NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science; Bachelor of Science in Applied Mathematics and Statistics			
QUALIFICATION CODE:	07BSOC; 07BAMS	LEVEL:	5
COURSE CODE:	LIA502S	COURSE CODE:	LINEAR ALGEBRA 1
SESSION:	JUNE 2022	PAPER:	THEORY
DURATION:	3 HOURS	MARKS:	100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER:	DR. DSI IIYAMBO
MODERATOR:	DR. N CHERE

INSTRUCTIONS

1. Attempt all the questions in the booklet provided.

2. Show clearly all the steps used in the calculations.

3. All written work must be done in black or blue inked, and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Question 1

Consider the vectors $\mathbf{p} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{q} = \mathbf{i} - 3\mathbf{j} + 12\mathbf{k}$

a) Find the unit vector in the direction of $\mathbf{p}$. [3]

b) Find the angle (*in degrees*) between $\mathbf{p}$ and $\mathbf{q}$. Give you answer correct to 1 d.p. [8]

Question 2

Consider the following matrices.

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{pmatrix}, \quad \text{and } D = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{pmatrix}.$$

a) Given that $C = AB$, determine the element c_{32}. [3]

b) Find $(3A)^T$. [3]

c) Is DB defined? If yes, then find it, and hence calculate $tr(DB)$. [7]

Question 3

Let $A = \begin{pmatrix} a_{ij} \end{pmatrix}$ be an $n \times n$ matrix.

a) When do we say that A is a symmetric matrix? [2]

b) Prove that $A + A^T$ is a symmetric matrix. [5]

c) Prove that if A is an invertible symmetric matrix, then A^{-1} is also symmetric. [6]

Question 4

Consider the matrix $A = \begin{pmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{pmatrix}$.

a) Use the *Cofactor expansion method* to evaluate the determinant of A. [7]

b) Is A invertible? If it is, find A^{-1}. [14]

c) Find $\det\left(3(2A)^{-1}\right)$. [6]

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Question 5

Use the *Gaussian elimination method* to find the solution of the following system of linear equations, if it exists.

$$x_1 + 3x_2 - x_3 = 1$$
$$2x_1 + x_2 + x_3 = 4$$
$$3x_1 + 4x_2 + 2x_3 = -1$$

[10]

Question 6

a) Prove that a vector space cannot have more than one zero vector. [6]

b) Let M_{nn} be a vector space whose elements are all the $n \times n$ matrices, with the usual addition and scalar multiplication for matrices. Determine whether the following set is a subspace of M_{nn}.

$$S = \{A \in M_{nn} \mid tr(A) = 0\}$$

[11]

c) Prove or disprove that if S and T are subspaces of a vector space V, then $S \cap T$ is also a subspace of V. [9]

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